Enrollment No: ______________________ **Exam Seat No:** ______________________

C.U.SHAH UNIVERSITY

Winter Examination- 2022

Subject Name: Differential Equations

Subject Code: 5SC01DIE1 **Branch: M.Sc. (Mathematics)**

Semester: 1 **Date: 03/01/2023** **Time: 11:00 To 02:00** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION-I

Q-1 **Attempt the following questions** **(07)**

a) What is the degree of differential equation $y'' + (y' - x)^{\frac{1}{2}} = 0$. **(01)**

b) Solve: $\frac{dy}{dx} = \frac{x-y}{x}$. **(01)**

c) Write down a form of Bessel's differential equation. **(01)**

d) Find the particular integral of the differential $(D-1)(D-2)y = e^{3x}$. **(01)**

e) Write down Lagrange's auxiliary equation form of the differential equation $(x-y)p + (y-z)q = (z-x)$. **(01)**

f) Solve: $\frac{dy}{dx} = (x+y)$. **(02)**

Q-2 **Attempt all questions** **(14)**

a) Find the power series solution of the differential equation $\frac{d^2y}{dx^2} + 3x\frac{dy}{dx}3y = 0$ about point $x = 0$. **(06)**

b) Discuss the singularities of the differential equation **(05)**

$$x^2(x^2+1)\frac{d^2y}{dx^2} + (x^2-1)\frac{dy}{dx} + 2y = 0.$$

c) Find the solution of $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = e^x$. **(03)**

OR

Q-2 **Attempt all questions** **(14)**

a) Find the power series solution of the differential equation $\frac{d^2y}{dx^2} + y = 0$ about point $x = 0$. **(05)**

b) Solve the differential equation $y'' + a^2y = cos\, cos\, ax$ by using the method of variation of parameters. **(05)**

c) Solve: $\frac{dy}{dx} + (tan\, tan\, x + \frac{1}{x})y = \frac{sec\, sec\, x}{x}$. **(04)**

Q-3 **Attempt all questions** **(14)**

a) Solve the differential equation $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=\frac{1}{x^2e^x}$ by using the method of variation parameters. **(07)**

b) If $y_1=e^{2x}$ and $y_2=xe^{2x}$ then find $W(y_1,y_2)$ at $x=0$ **(04)**

c) Solve $\frac{dy}{dx}=(x+y+1)$. **(03)**

OR

Q-3 **Attempt all questions** **(14)**

a) Solve: $(mz-xy)p+(nx-lz)q=ly-mx$. **(06)**

b) If $X=((yz+2x),(zx-2z),(xy-2y)$ then show that $Xcrul(X)=0$. **(05)**

c) Solve: $px+qy=z$ **(03)**

SECTION-II

Q-4 **Attempt the following questions** **(07)**

a) Solve: $\frac{d^2y}{dx^2}-y=0$. **(01)**

b) Solve: $\frac{dy}{dx}=x^2y+y$. **(02)**

c) Write down the polynomial $(3x^2-x-1)$ in terms of Legendre's equation. **(02)**

d) Verify that the equation $yzdx+xzdy+xydz=0$ is integrable or not? **(02)**

Q-5 **Attempt all questions** **(14)**

a) Solve: $(yz+xyz)dx+(zx+xyz)dy+(xy+xyz)dz=0$. **(05)**

b) Find the approximate solution by Picard's method to the initial value problem $\frac{dy}{dx}=x+y\quad y(0)=1$ to obtain a value of y for $x=0.1$ correct up to three decimal places. **(05)**

c) Solve: $(y+z)dx+dy+dz=0$. **(04)**

OR

Q-5 **Attempt all questions** **(14)**

a) State and prove Rodrigue's formula. **(06)**

b) Prove that $(n+1)P_{n+1}(x)=(2n+1)xP_n(x)-nP_{n-1}(x)$, for all $n\geq 2$. **(04)**

c) State and prove Orthogonality of Bessel's function. **(04)**

Q-6 **Attempt all questions** **(14)**

a) In usual notation prove that $J_n(x)=\frac{1}{\pi}\int_0^{\pi}\quad cos\,cos\,(n\theta-xsin\theta)d\theta$ if $m=n$. **(05)**

b) Solve $z^2=pqxy$ using Charpit's method. **(05)**

c) Prove that the $J_n(x)$ and $J_{-n}(x)$ are linearly independent. **(04)**

OR

Q-6 **Attempt all questions** **(14)**

a) Find $\frac{c_6}{c_4}$ in power series solution of $y'=x^2-4x+1$ with $y(2)=3$. **(07)**

b) State and prove the Orthogonality of Legendre's polynomials. **(07)**